

A digital approach in eliminating the higher order systematic effects in Ptolemy's *Geographia* longitude and latitude differences

Keywords: Ptolemy's *Geographia*; Ptolemy's coordinates; Ptolemaic reference system; best fitting; spatial distribution of longitude and latitude differences.

Summary

In his celebrated *Geography*, Claudius Ptolemy describes geographic sites (i.e. towns, mountain peaks, river mouths, promontories and other) as points with given coordinates of spherical longitude and latitude type. These geographic coordinates are following the known Ptolemaic reference system of parallels and meridians, the origin of which are respectively close to actual Equator and close to the Canary Islands many degrees west of the today's origin at Greenwich. It is also known that though latitudes are rather well defined, considering the level of measuring accuracy at Ptolemy's times, the longitudes suffer severe shortcomings which are due to the difficulties of measurement time, which corresponds to the longitude. The longitude values given by Ptolemy are also strongly dependent upon the distance from the Canaries eastwards. Previous research showed the order of magnitude of the longitude and latitude differences of Ptolemy's values from the today's counterparts both in broader and local scale, but without diving into a systematic geodetic approach on the issue. The problem is challenging and deserves a revisiting because of the advances of digital computational and visualization technologies (informatics and infographics) which are massively available today allowing new approaches and techniques in studying this extraordinary document of our cartographic heritage as it is the Ptolemy's *Geographia*. In the paper, part of a broader research carried out the last years by the Cartography Group of our Faculty we focus our interest on Ptolemy's coordinates given in *Geography* for the area of actual Greece, listed mainly in Book III, Chapter XII about Europe and Table X, but also in some other associated Chapters and Tables about Greece. Storing digitally the coordinates for the area of interest (almost 600 pairs of coordinates), and snooping the data, which is a laborious process because it requires the cross-checking with the relevant coordinated given in a number of Ptolemy's *Geographia* editions (in our case there are used five), the finally accepted list is formed which is compared with their today's values. The core of the study concerns a two-dimensional spatial analysis of the field of differences, testing various transformation functions in order to determine and eliminate the systematic error pattern inherent in Ptolemy's coordinates. The result, using new "reductive" methods in the comparison analysis (e.g. the concepts of the unit sphere, of the common projective support) with all affined illustrations of the associated test, shows the pattern of coordinate differences free of systematic effects up to the 2nd order, testing also and some higher order effects in order to get a better understanding of the whole process. Finally a field of various classes of spatial deformations of isotropic and anisotropic character, are for the first time tested and visualized.

Introduction

The interest in the geometric properties of historic maps has never been exhaustively and continuously treated by analytical means, especially in the modern era of cartography.

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The analytical treatment of the geometric background of early maps is an issue that today attracts the attention it deserves, as a result of the challenging perspectives opened by new digital technologies. These new technologies offer generously adequate processing tools that allow diving into the world of the geometric origin and properties of historic cartographic representations and maps.

The core of this study concerns a two-dimensional spatial analysis of the field of differences, testing various transformation functions in order to determine and eliminate the systematic error pattern inherent in Ptolemy's coordinates. The result, using 'reduction' methods in the comparison analysis (e.g. the concepts of the unit sphere, of the common projective support) with all affined illustrations of the associated test, shows the pattern of coordinate differences free of systematic effects up to the second order.

Ptolemaic reference system and coordinates

Ptolemy, in his *Geographia*, gives a list of geographic coordinates of spherical longitude and latitude of almost ten thousand of point locations, on the earth surface, as known at his times. These points are referred to geographic sites (i.e. towns, mountain picks, river mouths, promontories and other) and their geographic coordinates are following the known Ptolemaic reference system of parallels and meridians, the origin of which are respectively close to actual Equator and close to the Canary Islands almost 25 degrees west of the today's origin at Greenwich (Figure 1).

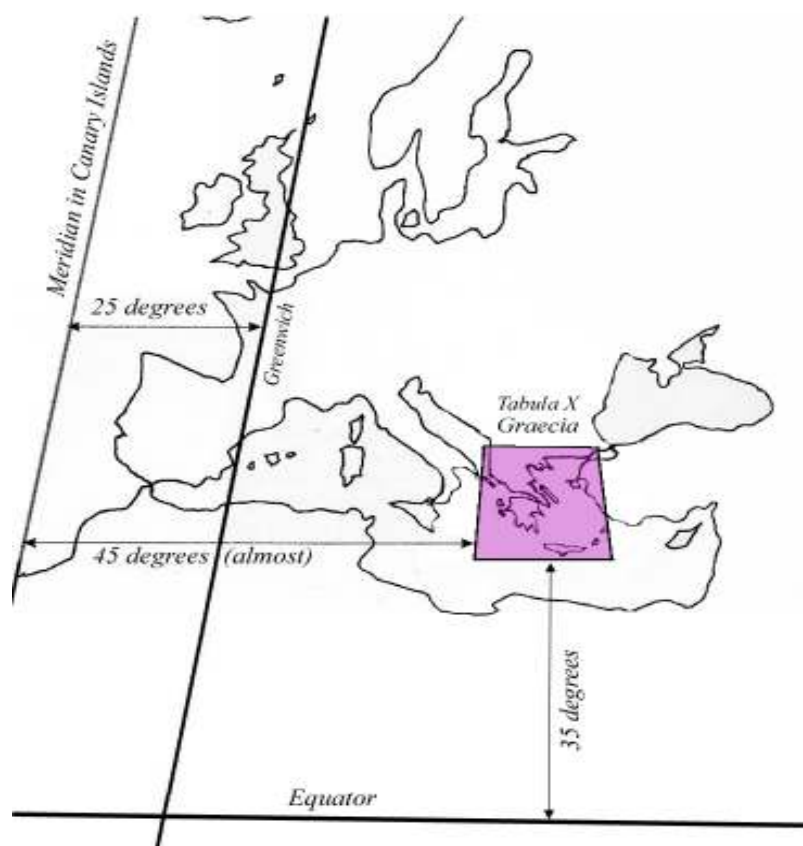


Figure 1. The origin of parallels and meridians in Ptolemy's *Geographia*

Coordinates in Ptolemy's *Geographia*

In this paper, which is a part of a broader research carried out the last years by the Cartography Group in the Faculty of Surveying Engineering at Thessaloniki, we focus our interest on Ptolemy's coordinates given in *Geographia* for the area of actual Greece, listed mainly in Book III, Chapters XI to XV concerning Europe and Book V and Chapter II concerning Asia. The tables referred to these chapters are Tables IX and X of Europe and Table I of Asia, as it is shown in the Table below (Table 1), with the largest part of actual Greece depicted in Table X (Figure 2). As it is obvious from the second figure, the world of Ptolemy is classified in Regions, since each chapter is referred to one of them, giving by this way the Atlas concept. The smaller the table is the more important and detailed the region appears to be in Ptolemy's *Geographia*.

Book III		Tables
Thrace	Chapter XI	Ninth map of Europe
Macedonia	Chapter XII	Tenth map of Europe
Epirus	Chapter XIII	
Achaia	Chapter XIV	
island Crete	Chapter XV	
Book V		
Asia properly so called (Asia Minor)	Chapter II	First map of Asia

Table 1. Chapters and tables referred to Greece in Ptolemy's *Geographia*



Figure 2. The 'Tabulae' in Ptolemy's *Geographia*.

The digital process

According to the procedure we follow, we store, at the beginning, digitally the coordinates for the area of interest from the different editions of Ptolemy's *Geographia* we have, in a database. In this case we are talking about almost 800 pairs of coordinates, 600 of them refer to the actual territory of Greece. The editions, we use for this particular study are the following five:

- the *Donnus Nicolaus Germanus* mid-15th century manuscript of Ptolemy's *Geographia* as given in Codex Ebnerianus (Stevenson 1991: 92),
- two 19th century editions by *Nobbe* (Leipzig 1843, 1966),
- the *Müller's* edition (Paris 1883),
- the *Vatopedion Codex* (13th -14th century) and
- the *Marciana Codex* (15th century).

Processing the Ptolemy's Coordinates

The coordinates from the five sources, are independently and mutually checked and evaluated in order to detect discrepancies in the point placement, and then projected onto a map with a relevant graticule of parallels and meridians, all of them plotted in the same projection, e.g the elementary geographic projection ($y = R\phi, x = R\lambda$), assuming a unit radius reference sphere ($R=1$) for the earth's model. In this process maps are plotted from the coordinates and by this way the locations of points are visualized, making easier the auto- and cross- checking of the values, the detection of the differences, the gross errors, the double values and other displacements they may occur. In Figure 3 one of the

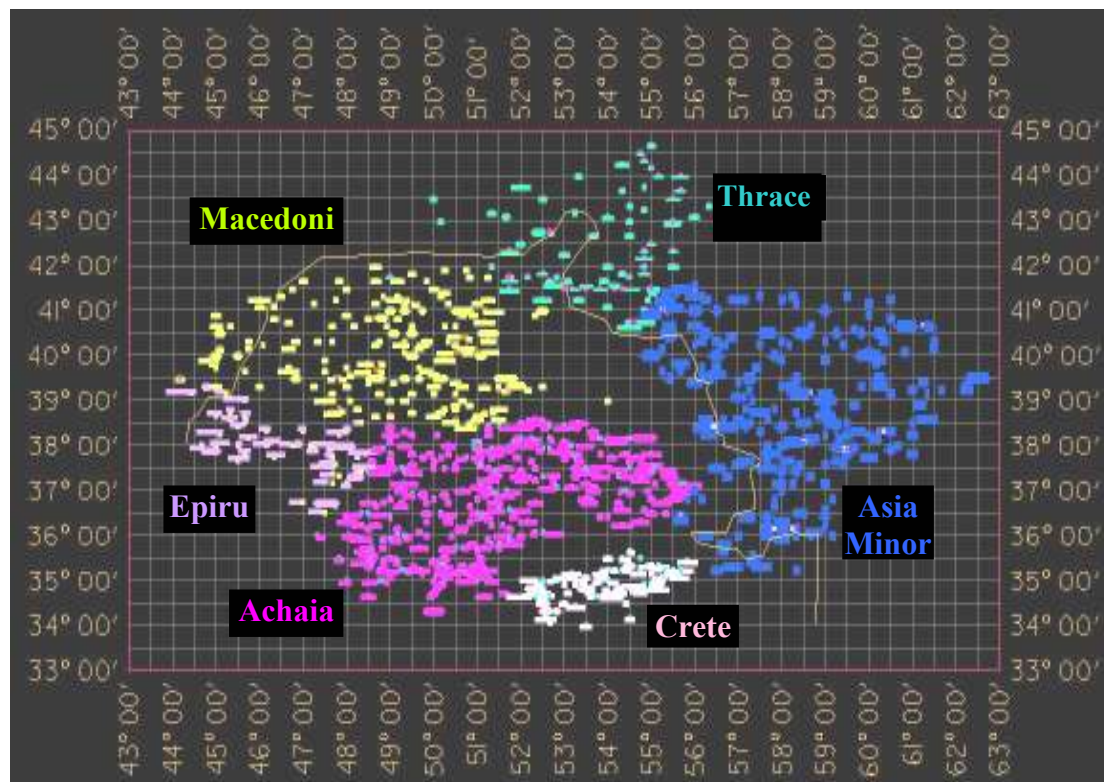


Figure 3. Coordinate plotting in geographic projection according to the snooped list of Ptolemy's coordinates and actual borders of Greece. The point colours refer to the different regions, as defined by Ptolemy.

these maps is shown, in which the toponyms are depicted concerning the regions that are referred totally or partially to the actual territory of Greece. The yellow line shows in an approximate way, the actual borders of modern Greece, while the different colours of the points are referred to the different regions as described in Ptolemy's *Geographia*.

Having concluded to an 'accepted' list of coordinates without gross errors, double values or records or other apparently erroneous discrepancies in point placement, we start the comparison of Ptolemy's coordinates with their today's counterparts. In order to perform such a comparison and to identify the coincidence of places in Ptolemy's era with their today's counterparts, we had to compare the toponyms of each area with the toponyms of the corresponding area of actual territory of Greece confirming at the end the coincidence of the with certainty known points in both cases, based mainly on relevant references in historical and archeological sources. In the map of Figure 4, we can see on a modern map, the places where some of Ptolemy's toponyms are detected according to historical and other sources.

Best fitting of Ptolemy's representation to a modern map

The points, we have mentioned before, have great importance to the continuity of this work because a set of them, properly distributed to the overall map space, is selected and brought into one to one correspondence with the actual coordinates of the same set of points in the modern map, after choosing a transformation system, in this case a 2nd order polynomial transformation, involving a projection and an earth's model. The result of the best fitting of Ptolemy's coordinates to the modern counterparts is shown in Figure 5. The Ptolemy representation is georeferenced to actual geographic coordinates using

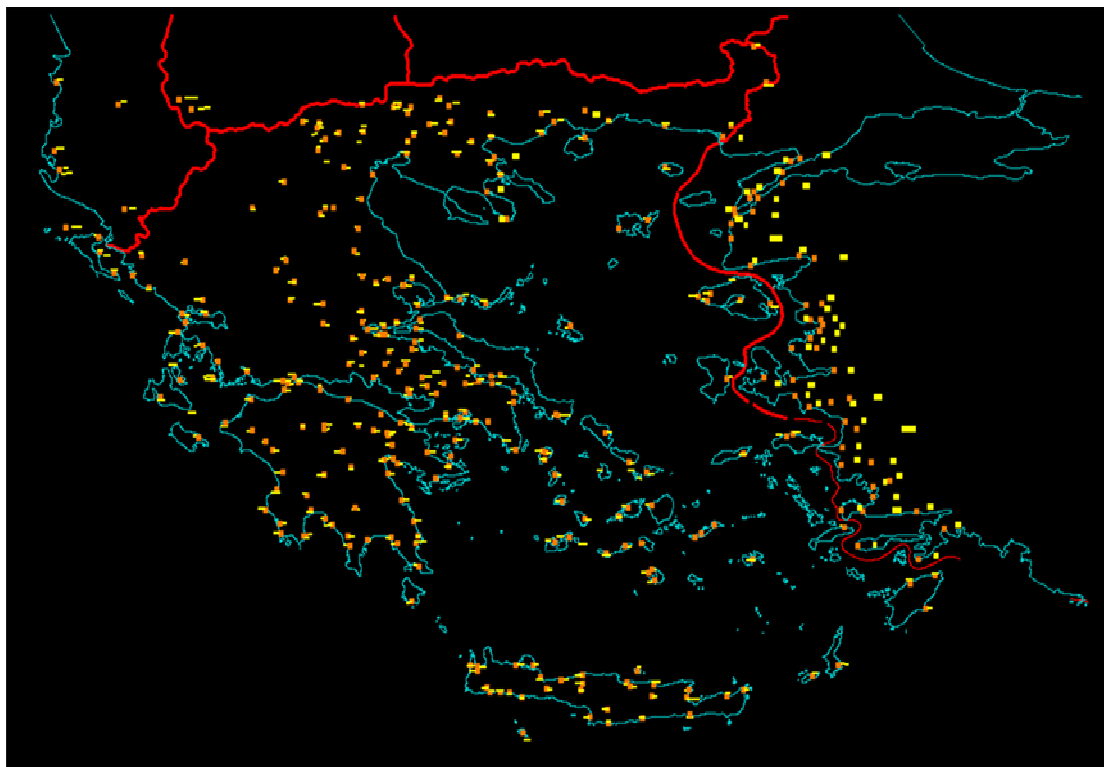


Figure 4. Ptolemy's toponyms on a modern map

almost 350 control points properly distributed in the area of Greece. Ptolemy's graticule, extended from 43° to 63° in longitude and from 33° to 45° in latitude, contrary to the geographic graticule of the modern map, which extended from 18° to 30° in longitude and from 33° to 44° in latitude. In the resulting map (Figure 5), Ptolemy's map of coordinates transforms into the actual coordinates and the deformation appeared in Ptolemy's graticule is obvious.

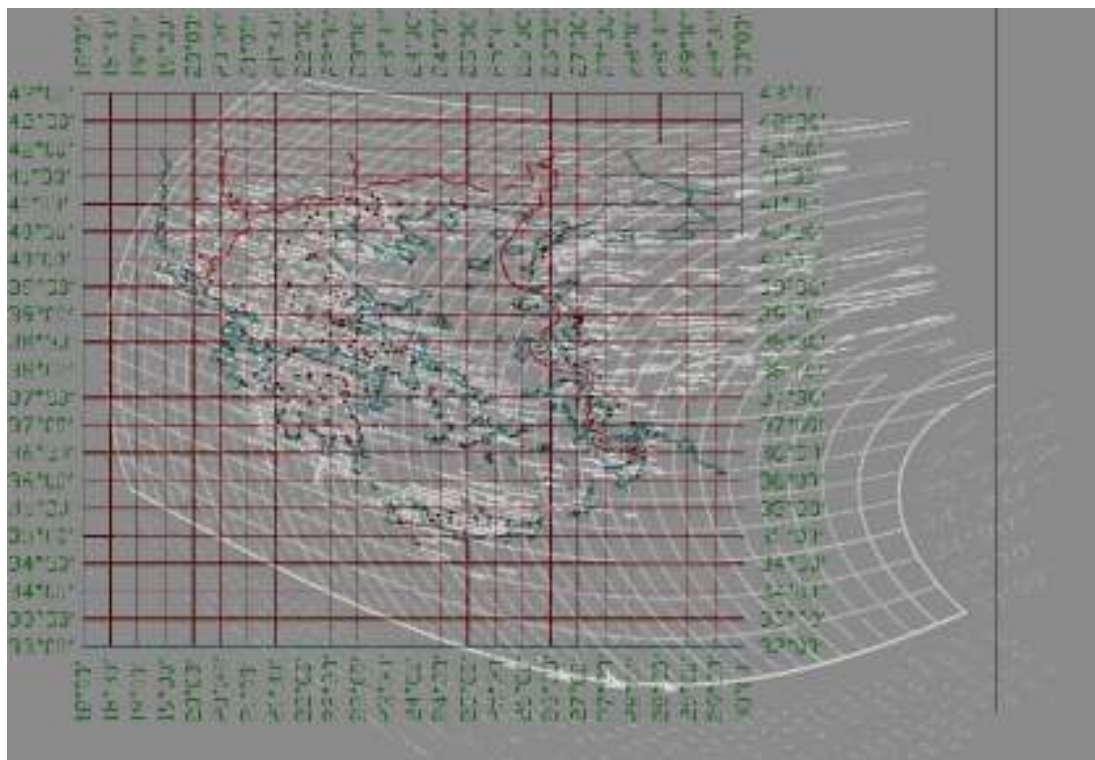


Figure 5. Second order polynomial best fitting of Ptolemy's representation in to a modern relevant map

The spatial distribution of differences in longitude and latitude

Previous research showed the order of magnitude of the longitude and latitude differences of Ptolemy's values from the today's counterparts both in broader and local scale, but without diving into a systematic geodetic approach on the issue. The problem is challenging and deserves a revisiting because of the advances of digital computational and visualization technologies (informatics and infographics) which are massively available today allowing new approaches and techniques in studying this extraordinary document of our cartographic heritage as it is the Ptolemy's *Geographia*.

Using the best fitting of Ptolemy's representation to the modern map, we study also, the spatial distribution of the differences in longitude and latitude induced after the comparison of Ptolemy's coordinates with their actual values. In the next two figures, which depict the distribution of the differences in both cases (Figure 6 and 7), it is obvious that the distribution is not the same in both cases.

As we can see below, the longitude differences vary from 25° at the west side of Greece to 31° at the east side whereas the latitude differences are of much smaller magnitude than those of longitude and vary from -2° West to almost 1° East. These differences can

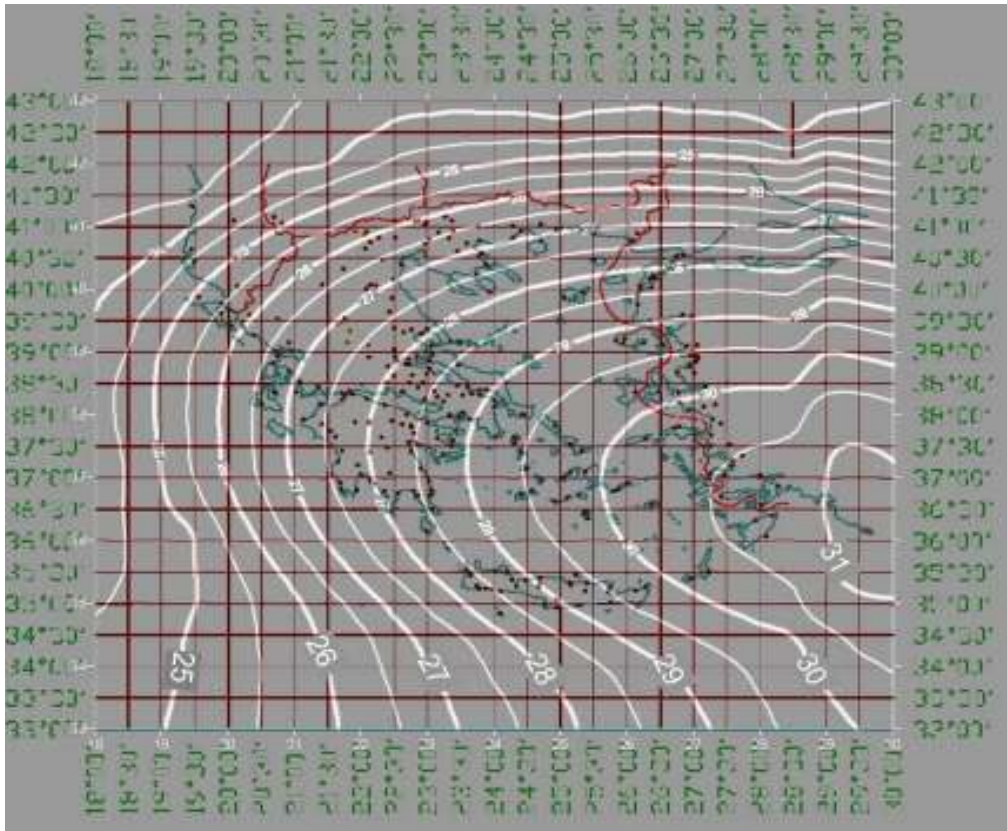


Figure 6. The isolines of longitude differences, in degrees, between Ptolemy's values and their actual counterparts

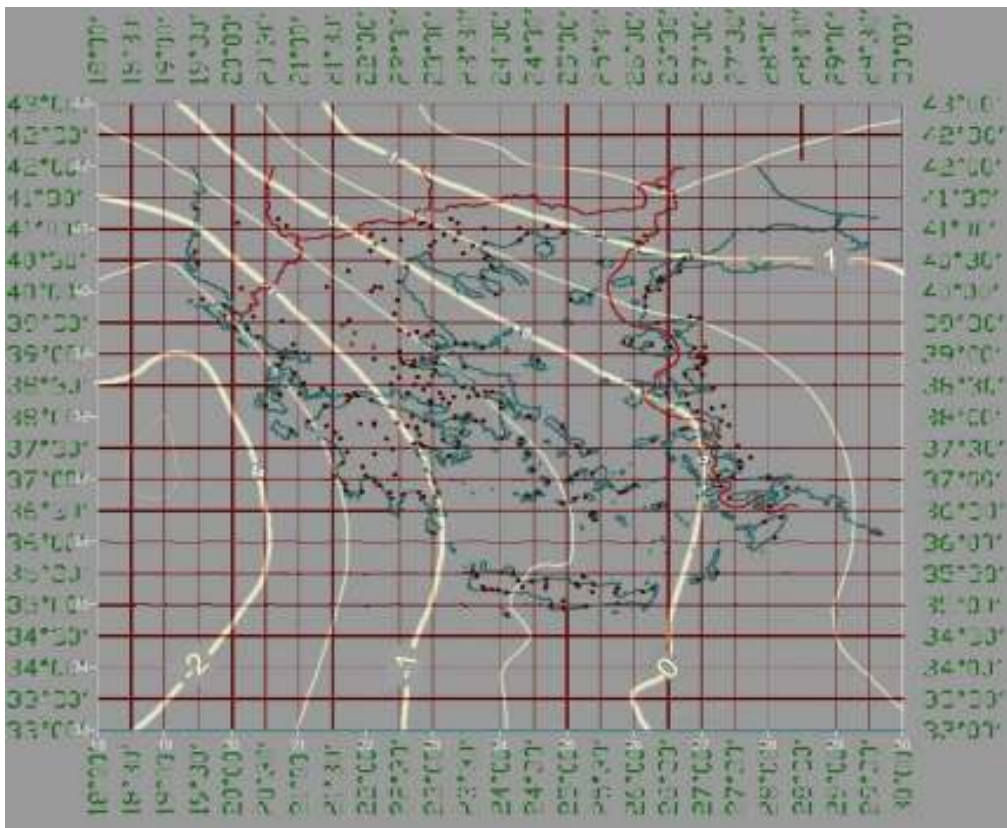


Figure 7. The isolines of latitude differences, in degrees, between Ptolemy's values and their actual counterparts

be easily explained by the fact that though latitudes are rather well defined, considering the level of measuring accuracy at Ptolemy's times, the longitudes suffered severe shortcomings which are due to the difficulties in measuring the time, which corresponds directly to longitude. The longitude values given by Ptolemy are also strongly dependent upon the distance from the Canaries eastwards.

Concluding remarks

The transformation of early maps into digital form and their comparison with modern maps using new processing methods and technologies is of great importance for the study of the geometric properties of early cartographic documents. Best fitting techniques are appropriate in order to compare early cartographic representations with their modern counterparts.

In this study particularly, the result of the two-dimensional spatial analysis of the field of differences in Ptolemy's coordinates shows the pattern of coordinate differences free of systematic effects up to the 2nd order. This work is extended by testing also and some higher order effects in order to get a better understanding of the whole process.

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